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We analyze a finite horizon periodic review joint pricing and inventory management model for a firm that replenishes and sells a product under the *scarcity effect* of inventory. The demand distribution in each period depends negatively on the sales price and customer-accessible inventory level at the beginning of the period. The firm can withhold or dispose of its on-hand inventory to deal with the scarcity effect. We show that a customer-accessible-inventory-dependent order-up-to/dispose-down-to/display-up-to list-price policy is optimal. Moreover, the optimal order-up-to/display-up-to and list-price levels are decreasing in the customer-accessible inventory level. When the scarcity effect of inventory is sufficiently strong, the firm should display no positive inventory and deliberately make every customer wait. The analysis of two important special cases wherein the firm cannot withhold (or dispose of) inventory delivers sharper insights showing that the inventory-dependent demand drives both optimal prices and order-up-to levels down. In addition, we demonstrate that an increase in the operational flexibility (e.g., a higher salvage value or the inventory withholding opportunity) mitigates the demand loss caused by high excess inventory and increases the optimal order-up-to levels and sales prices. We also generalize our model by incorporating responsive inventory reallocation after demand realizes. Finally, we perform extensive numerical studies to demonstrate that both the profit loss of ignoring the scarcity effect and the value of dynamic pricing under the scarcity effect are significant.

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1. Introduction

In the operations management literature, joint pricing and inventory management has received extensive attention. A key assumption in existing models in this stream of literature is that demand, though random, is independent of inventory (e.g., Fedegruen and Heching 1999), so that sales and, hence, revenue are linked to inventory only through the stockout effect.

In quite a few industries (e.g., automobile, electronics, and luxury products, etc.), however, we have observed strong empirical and anecdotal evidence that demand may be correlated with the amount of inventory carried by retailers. A high inventory level sometimes promotes sales because it creates a strong visual impact (the *billboard effect*) and signals abundant potential availability, both of which can make the item more desirable and increase the chance of customer purchase. On the other hand, it is also commonly observed in practice that an ample inventory conveys to the customers the message that the item is of low popularity and quality, thus inducing low demand.

The negative correlations between demand and inventory are well supported by psychological and economic theories as well as rich anecdotal observations and empirical data. The phenomena that a low inventory level may increase and a high inventory level may decrease demand are often

referred to as the scarcity effect of inventory. Three major mechanisms drive this effect: (1) inventory level signals the quality and popularity of a product; (2) inventory level implies stockout risk of a product; and (3) inventory level reveals the pricing strategy the retailer will use. We now discuss these three mechanisms in detail.

First, it has been well established in psychological commodity theory that supply scarcity increases the attractiveness of a product to customers (Brock 1968). This notion has been tested and refined by various experiments with respect to a wide variety of product categories (e.g., food, wine, and books) by, e.g., Worchel et al. (1975), Verhallen and Robben (1994) and van Herpen et al. (2009). The desirability of the product is enhanced by scarce inventory because customers are likely to infer product quality and popularity from its inventory level. A lower inventory level signals more consumption by other customers and, hence, that the product is more popular and of higher quality. On the other hand, observing a high inventory, a customer naturally believes that there are many units on hand because no one wants to buy the item. Some recent marketing (e.g., Stock and Balachander 2005) and operations management (e.g., Veeraraghavan and Debo 2009) papers use game-theoretic models to demonstrate that the scarcity strategy can effectively signal to the customers the high quality of a

product, thus creating a "hot product." Empirical results on the scarcity effect of inventory on demand in the automobile industry can also be found in, e.g., Balachander et al. (2009) and Cachon et al. (2013).

Second, a low inventory level spreads a sense of urgency among customers that soon the product will be sold out and potential buyers will be put on a wait list. Such backlogging risk motivates customers to make an immediate purchase instead of searching for better options. A high inventory, however, grants customers the luxury of waiting and searching, thus lowering the current demand. A similar mechanism also drives the search behavior that a low inventory of one product type discourages a customer from searching for better types (Cachon et al. 2008). Knowingly limiting the availability of a product, the retailer can induce "buying frenzies" among uninformed customers and set a higher price (DeGraba 1995).

Third, as shown in pricing and revenue management literature (e.g., Federgruen and Heching 1999, Gallego and van Ryzin 1994), retailers increase their sales prices when inventories are low. Therefore, customers infer from a low inventory level that it is unreasonable to expect a lower price and decide to purchase the item immediately (see, e.g., Aviv and Pazgal 2008). On the other hand, a high inventory level suggests that the sales price will be more likely to decrease; this encourages customers to wait before buying. Carefully making use of this mechanism, the retailer can enjoy the benefits of inducing customers to purchase early at high prices (Liu and van Ryzin 2008). A similar idea has also been adopted in the advance selling literature (e.g., Xie and Shugan 2001), which shows that firms may limit their capacity for advance selling to credibly signal their pricing strategy to customers.

Along with the rich theoretical and empirical justifications of the scarcity effect of inventory, practitioners have extensively adopted this idea in their marketing strategies. Dye (2000) and Brown (2001) document that the scarcity strategy, in which product supply is deliberately limited, has become a basic tactic for marketers to promote sales. An increasing number of automobile manufacturers create significant levels of scarcity and make a long list of hard-to-get car models over time (see Balachander et al. 2009). Although facing thousands of customers on the wait lists, none of the manufacturers rushed to accelerate production (*Wall Street Journal* 1999). Likewise, Maynard (2006) shows that BMW promotes its Mini Cooper line by limiting supply and letting potential owners wait for, on average, two and half months for the models. The limited distribution strategy has accelerated demand for the Mini Cooper since its reintroduction in the U.S. market. A similar promotional strategy is also used in the electronics market, especially at the introduction stage of a new product generation; fans have been excited by the long wait for Sony Play Stations (*Retailing Today* 2000), Nintendo Game Boys (*Wall Street Journal* 1989), and Apple iPads 2 (Sherman 2010).

In this paper, we study the dynamic pricing and inventory management model under the scarcity effect of inventory. The stochastic demand is modeled as a decreasing function of the sales price and the customer-accessible inventory level at the beginning of each decision epoch. Unmet demand is fully backlogged to the next period. The wait lists observed or spread through word-of-mouth successfully signal the high quality and popularity of the product and attract more customers (see, e.g., Brown 2001 and Dye 2000). From the strategic perspective, joint pricing and inventory decisions effectively deliver information about the quality and popularity of the product. Specifically, pricing flexibility induces more strategic customer behavior (e.g., waiting for potential price discount), which further strengthens the scarcity effect of inventory because customers may anticipate price changes based on current inventory (see, e.g., Liu and van Ryzin 2008).

We develop a unified joint price and inventory management model that incorporates both inventory withholding and inventory disposal to address the scarcity effect. Under the inventory withholding policy, the firm displays only part of its inventory and withholds the rest in a warehouse not observable by customers; this induces higher potential demand. Analogously, with inventory disposal, the firm can dispose of unnecessary excess inventory that has some salvage value. Both inventory withholding and disposal may incur a cost. We show that a customer-accessible-inventory-dependent order-up-to/dispose-down-to/display-up-to list-price policy is optimal. Moreover, the order-up-to/display-up-to and list-price levels are decreasing at the customer-accessible inventory level. When the scarcity effect of inventory is sufficiently strong, the firm should display no positive inventory so that every customer must wait before getting the product. In this case, the strong scarcity effect creates more opportunities than risks, so the firm can proactively take advantage of it and induce more demand by making customers wait (e.g., the BMW marketing strategy).

When it is too costly to withhold or dispose of inventory, the unified model is reduced to the model without inventory withholding or the model without inventory disposal, both of which deliver sharper insights. In the model without inventory withholding/disposal, we show that the inventory-dependent demand increases the overstocking risk and, thus, lowers optimal sales prices and order-up-to levels. With higher operational flexibility (a higher salvage value or the inventory withholding opportunity), however, the firm addresses the scarcity effect of inventory more effectively and, hence, increases its sales prices and order-up-to/display-up-to levels. In short, inventory disposal/withholding benefits the firm by enhancing its operational flexibility and agility.

We also generalize the unified model by incorporating responsive inventory reallocation, which allows the firm to reallocate (with cost) its inventory between display and warehouse after demand is realized.. In this case, the firm

can keep a low inventory and better hedge against risks of demand uncertainty and the scarcity effect of inventory.

We perform extensive numerical studies to demonstrate (a) the robustness of our analytical results, (b) the impact of the scarcity effect on the profitability of the firm, and (c) the value of dynamic pricing under the scarcity effect of inventory. Our numerical results show that the analytical characterizations of the optimal policies in our model are robust and hold in all of our numerical experiments. Both the profit loss of ignoring the scarcity effect and the value of dynamic pricing under the scarcity effect are significant, and increase the intensity of the scarcity effect and/or demand variability. The reasons are: (1) the scarcity effect decreases future demand and magnifies future demand variability; and (2) dynamic pricing allows the firm to induce higher future demand and dampen future demand variability. In addition, a longer planning horizon increases the impact of the scarcity effect and decreases the value of dynamic pricing.

We summarize the main contributions of this paper as follows: (1) To our knowledge, we are the first to study joint pricing and inventory management under the scarcity effect of inventory. We characterize the optimal policy in a general unified model and generalize our results to the model with responsive inventory reallocation. (2) We analyze the impact of the scarcity effect of inventory on the firm's optimal price and inventory policies and study the effect of operational flexibilities on the firm's optimal decisions under the scarcity effect. (3) We identify the rationale behind the phenomenon that firms with intense scarcity effect deliberately make their customers wait before getting the product. (4) We numerically study the profit loss of ignoring the scarcity effect and the value of dynamic pricing under this effect.

The rest of the paper is organized as follows. In §2, we position this paper in the related literature. Section 3 presents the basic formulation, notations, and assumptions of our model. In Section 4, we propose and analyze the unified model. Section 5 discusses additional results and insights in two important cases, i.e., the model without inventory withholding and the model without inventory disposal. Section 6 generalizes the unified model to the model with responsive reallocation. Section 7 reports our numerical findings. We conclude by summarizing our findings and discussing a possible extension in §8. All proofs are relegated to the online appendix (available as supplemental material at http://dx.doi.org/10.1287/opre.2014.1306).

2. Literature Review

This paper is mainly related to two lines of research in the literature, i.e., (1) inventory management with inventory-dependent demand, and (2) optimal joint pricing and inventory policy.

There is a large body of literature on inventory-dependent demand. We refer interested readers to Urban (2005) for

a comprehensive review. Demand dependence on inventory is usually modeled in two ways in the literature, i.e., (1) potential demand is increasing in the inventory level after replenishment, and (2) potential demand is decreasing in the inventory level before replenishment (i.e., leftover inventory from the previous period).

The first approach to model inventory-dependent demand assumes that demand increases with inventory (the *billboard effect*). Gerchak and Wang (1994) study a periodic review inventory model in which the random demand in each period is increasing in the inventory level after replenishment. Dana and Petruzzi (2001) consider a single-period newsvendor model where demand is decreasing in price and positively correlated with inventory level. Several other operations management and marketing papers also assume that demand depends on the instantaneous (after replenishment) inventory level, in particular via the shelf-space effect. We refer interested readers to, e.g., Wang and Gerchak (2001 and 2002), Balakrishnan et al. (2004 and 2008), Martńez-de-Albéniz and Roels (2011), Baron et al. (2011) and Chen et al. (2012).

The other effect of inventory on demand, as discussed in §1, is the scarcity effect. That is, high leftover inventory (i.e., inventory at the beginning of the period before replenishment) negatively influences the potential demand. In the psychological commodity theory literature, Brock (1968) argues that supply scarcity increases the attractiveness of a product; this has been tested by numerous experiments in, e.g., Worchel et al. (1975) and van Herpen et al. (2009). Stock and Balachander (2005) and Veeraraghavan and Debo (2009) use game theoretic models to show that the firm can use the scarcity strategy to signal the high quality of a product. Aviv and Pazgal (2008), among others, demonstrate that customers may strategically wait for price discounts when observing a high inventory. Liu and van Ryzin (2008) propose an effective pricing scheme to induce customers to make early purchases under a revenue management framework. The idea that supply condition can signal potential pricing strategy and product quality has also been adopted in the advance selling literature (e.g., Xie and Shugan 2001 and Yu et al. 2014). Balachander et al. (2009) and Cachon et al. (2013) conduct empirical studies to show that the scarcity effect of inventory on demand prevails in the automobile industry.

To our knowledge, Sapra et al. (2010) is the only paper in the inventory management literature that incorporates the scarcity effect of inventory (called the "wait-list effect" in that paper) and assumes that potential demand is a decreasing function of leftover inventory. They show the optimality of understocking and propose the inventory withholding strategy. Our paper generalizes Sapra et al. (2010) in the following aspects: (1) We introduce a unified model that encompasses dynamic pricing, inventory withholding, and inventory disposal, and explicitly captures the interaction between price, inventory and demand. In particular, we analytically show the impact of inventory-dependent demand

on the firm's pricing policy, whereas Sapra et al. (2010) do not allow price adjustment during the planning horizon and numerically test the improvement of inventory-withholding policy under different price elasticities of demand. We also numerically show that the value of dynamic pricing under the scarcity effect of inventory is significant and increases with the scarcity effect intensity and/or demand variability. (2) Because of the endogenous pricing decision introduced to the dynamic program, analysis of our model is more involved and requires a different approach. (3) Two special cases of our unified model (i.e., the model without inventory withholding and the model without inventory disposal) demonstrate that inventory withholding and inventory disposal help mitigate the overage risk of inventory-dependent demand. (4) In addition to the understocking and inventory withholding policy proposed in Sapra et al. (2010), our model suggests three other strategies to dampen the negative effect of inventory-dependent demand: (a) price reduction, (b) inventory disposal, and (c) responsive inventory reallocation. (5) We show that when the scarcity effect of inventory is sufficiently strong, the firm should display no positive inventory and let customers wait. To summarize, this paper generalizes the model in Sapra et al. (2010) and strengthens its results and insights.

There is an extensive literature on dynamic pricing and inventory control under general stochastic demand. Federgruen and Heching (1999) study the inventory system in a periodic review model where the firm faces price-dependent demand in each decision period and unsatisfied demand is fully backlogged. A list-price order-up-to policy is shown to be optimal. This line of literature has grown rapidly since Federgruen and Heching (1999). For example, Chen and Simchi-Levi (2004a, b and 2006) analyze the joint pricing and inventory control problem with fixed ordering cost and show the optimality of (s, S, p) policy for finite horizon, infinite horizon, and continuous review models. Chen et al. (2006) and Song et al. (2009) study the joint pricing and inventory control problem under lost sales. In the case of a single unreliable supplier, Li and Zheng (2006) and Feng (2010) show that supply uncertainty drives the firm to charge higher prices under random yield and random capacity, respectively. Chen et al. (2011) take into consideration costly price adjustments in joint pricing and inventory management. When the replenishment lead time is positive, the joint pricing and inventory control problem under periodic review is extremely difficult. Pang et al. (2012) partially characterize the structure of the optimal policy. We refer interested readers to Chen and Simchi-Levi (2012) for a comprehensive survey on joint pricing and inventory control models. The major difference between our paper and this stream of research is that we take into account inventory-dependent demand and show that the scarcity effect of inventory drives the firm to order less, dispose more, withhold inventory, and charge a lower price. To our knowledge, only Dana and

Petruzzi (2001) and Balakrishnan et al. (2008) have studied the joint pricing and inventory control problem with inventory-dependent demand. However, both papers consider a single period model where demand is increasing in the available inventory after replenishment.

3. Model Formulation

We specify our unified model, notations, and assumptions in this section. Consider a firm facing random demand and periodically making pricing and inventory decisions in a T-period planning horizon, labeled backwards as $\{T, T-1, \ldots, 1\}$. The firm stores its on-hand inventory in two locations, one with customer-accessible inventory to satisfy and stimulate demand, and the other as a warehouse to withhold inventory that is unobservable to customers. The firm can replenish or dispose of inventory; it can also reallocate its on-hand inventory between the customer-accessible storage and the warehouse. If the firm places an order, the replenished inventory is delivered to the warehouse after which the firm decides how much inventory to reallocate to the customer-accessible storage. On the other hand, if the firm disposes of its on-hand inventory, it first ships inventory, if any, from the customer-accessible storage to the warehouse, and then chooses the disposal quantity.

In each period, the sequence of events unfolds as follows: At the beginning of each period, the firm reviews its total and customer-accessible leftover inventories from last period, simultaneously chooses the order/disposal and reallocation quantities and the sales price, pays the ordering and reallocation costs, and receives the disposal salvages. The ordering and reallocation lead times are assumed to be zero so that the replenished and reallocated inventories are received immediately. Inventory disposal is also executed at once. The demand is then realized and the revenue is collected. At the end of the decision period, the holding and backlogging costs are paid, and the total and customer-accessible inventories are carried over to the beginning of the next period.

The state of the system is given by:

$I_t^a =$ the starting customer-accessible inventory level before replenishment/disposal/reallocation in period t, $t = T, T-1, \ldots, 1$, where the superscript "a" refers to "customer-accessible";

$I_t =$ the starting total inventory level before replenishment/disposal/reallocation in period t, $t = T, T-1, \ldots, 1$.

Note that the amount of inventory the firm withholds in the warehouse is $I_t - I_t^a \geqslant 0$. We introduce the following notation to denote the decisions of the firm:

$p_t =$ the sales price charged in period t, $t = T, T-1, \ldots, 1$;
$x_t^a =$ the customer-accessible inventory level after replenishment/disposal/reallocation but before demand is realized in period t, $t = T, T-1, \ldots, 1$;

$x_t =$ the total inventory level after replenishment/disposal/reallocation but before demand is realized in period t, $t = T, T - 1, \ldots, 1$.

We assume that the price p_t is bounded from above by the maximum allowable price $\bar{p}$ and from below by the minimum allowable price $\underline{p}$. Without loss of generality, we also assume that the customer-accessible inventory storage capacity of the firm is K_a ($0 < K_a \leqslant +\infty$), whereas the warehouse capacity is infinite. In other words, the customer-accessible inventory level after replenishment/disposal/reallocation cannot exceed K_a in each period, i.e., $x_t^a \leqslant K_a$ for all $t = T, T - 1, \ldots, 1$. Following the "no-artificial wait-list" notion (see Sapra et al. 2010), we assume that the firm cannot decrease its customer-accessible inventory level if a wait list already exists, i.e., $x_t \geqslant x_t^a \geqslant \min\{I_t^a, 0\}$.

We introduce the following model primitives:

$\alpha =$ discount factor of revenues and costs in future periods, $0 < \alpha \leqslant 1$;

$c =$ purchasing cost per unit ordered;

$s =$ salvage value per unit disposed;

$b =$ backlogging cost per unit backlogged at the end of a period;

$h_a =$ holding cost per unit stocked and accessible to customers at the end of a period;

$h_w =$ holding cost per unit stocked in the warehouse at the end of a period;

$r_d =$ unit reallocation fee from the warehouse to the customer-accessible storage;

$r_w =$ unit reallocation fee from the customer-accessible storage to the warehouse.

Without loss of generality, we assume that the following inequalities hold:

$b > (1 - \alpha)(r_d + c)$: the backlogging penalty is higher than the saving from delaying an order to the next period, so that the firm will not backlog all of its demand;

$c > s$: unit procurement cost dominates the unit salvage value;

$\underline{p} > \alpha(c + r_d) + b$: positive margin for backlogged demand.

Note that although we assume that the parameters and demand are stationary throughout the planning horizon, the structural results in this paper remain valid when the parameters and demand distributions are time-dependent.

As discussed in §1, we assume that demand in period t, D_t, depends negatively on the prevailing price and customer-accessible inventory level at the beginning of this period according to a general stochastic functional form: $D_t = \delta(p_t, I_t^a, \epsilon_t)$, where ϵ_t is a random term with a known continuous distribution and a connected support. $\delta(\cdot, \cdot, \epsilon_t)$ is a twice continuously differentiable function strictly decreasing in p_t and decreasing in I_t^a for any ϵ_t. We

base our analysis of the problem on the following demand form:

$$\delta(p_t, I_t, \epsilon_t) = (d(p_t) + \gamma(I_t^a))\epsilon_t^m + \epsilon_t^a,$$
$$\text{where } \mathbb{E}\{\epsilon_t^a\} = 0 \text{ and } \mathbb{E}\{\epsilon_t^m\} = 1. \quad (1)$$

We assume that ϵ_t's are *i.i.d.* random vectors with ϵ_t^a supported on $[\underline{a}, \bar{a}]$ and ϵ_t^m supported on $[\underline{m}, \bar{m}]$ ($\underline{m} \geqslant 0$). At least one of the two random variables (ϵ_t^a and ϵ_t^m) follows a continuous distribution (i.e., $\underline{a} \neq \bar{a}$ or $\underline{m} \neq \bar{m}$). This ensures that D_t follows a nondegenerate continuous distribution supported on the interval: $[(d(p_t) + \gamma(I_t^a))\underline{m} + \underline{a}, (d(p_t) + \gamma(I_t^a))\bar{m} + \bar{a}]$ for any (p_t, I_t^a). Note that the above demand model is quite general and includes as special cases several demand models from the existing literature. For example, when $\epsilon_t^m = 1$ with probability 1, the demand model is reduced to the additive demand model; if $\epsilon_t^a = 0$ with probability 1, it is reduced to the multiplicative demand model (as a generalized version of the model proposed in Sapra et al. 2010); and if $\gamma(\cdot) \equiv 0$, the demand model is reduced to the standard price-dependent demand model (as proposed in Chen and Simchi-Levi 2004a). The term $d(p_t)$ summarizes the impact of price on demand in period t. As assumed above, $d(\cdot)$ is strictly decreasing in p_t. In some market where competition is fierce and the firm has no pricing power, the price is exogenously fixed at p_0 and the price induced demand is fixed at $d_0 = d(p_0)$. The term $\gamma(I_t^a)$, which is a decreasing function of I_t^a, captures the scarcity effect of inventory on demand. Hereafter, we refer to $\gamma(\cdot)$ as the scarcity function, and $\gamma'(\cdot)$ as the intensity of scarcity effect. The dependence of demand on inventory is measured by $\gamma'(\cdot)$. i.e., the smaller the $\gamma'(\cdot)$, the more intensive the potential demand depends on the customer-accessible inventory level. When demand is independent of inventory, $\gamma(I_t^a) \equiv \gamma_0$ for all customer-accessible inventory level I_t^a. Note that our demand model generalizes the model in Sapra et al. (2010) in the sense that our model also captures the impact of endogenous sales price on demand.

Since $d(\cdot)$ is strictly decreasing in p_t, we assume $p(d_t)$ to be its strictly decreasing inverse. For the convenience of our analysis, we change the decision variable from p_t to $d_t \in [\underline{d}, \bar{d}]$, where $\underline{d} = d(\bar{p})$ and $\bar{d} = d(\underline{p})$. To avoid the unrealistic case where demand becomes negative, we assume that $\underline{d} + \gamma(K_a) \geqslant 0$ to ensure that $\mathbb{E}\{D_t\} = d_t + \gamma(I_t^a) \geqslant 0$ for any $d_t \in [\underline{d}, \bar{d}]$ and $I_t^a \leqslant K_a$. We impose the following three assumptions throughout our analysis:

ASSUMPTION 1. *The inverse demand function $p(\cdot)$ is twice continuously differentiable and concavely decreasing in d_t, with $p'(d_t) < 0$ for $d_t \in [\underline{d}, \bar{d}]$. In addition, $p(d_t)d_t$ is concave in d_t.*

The concavity of $p(d_t)d_t$ in d_t suggests the decreasing marginal revenue with respect to demand, which is a standard assumption in joint pricing and inventory management literature, see, e.g., Chen and Simchi-Levi (2004a), Li and Zheng (2006) and Pang et al. (2012). For a more comprehensive discussion on decreasing marginal revenue

assumptions, see Ziya et al. (2004). The concavity of $p(\cdot)$ implies that the demand is more price sensitive when sales prices are higher. This is also a common assumption in the literature; see, e.g., Federgruen and Heching (1999).

As Sapra et al. (2010), we also assume that demand is concavely decreasing in the customer-accessible leftover inventory.

ASSUMPTION 2. *The scarcity function $\gamma(\cdot)$ is concavely decreasing and twice continuously differentiable. In addition,*

$$\lim_{I_t^a \to -\infty} \gamma'(I_t^a) = 0 \quad and \quad \lim_{I_t^a \to -\infty} \gamma(I_t^a) = \gamma_0.$$

The concavity of $\gamma(\cdot)$ refers to the phenomenon that a higher customer-accessible leftover inventory level has a greater marginal effect on potential demand. However, when the backlogged demand is very high, its value for stimulating high potential demand is limited because $\gamma(\cdot)$ is bounded from above. In other words, the impact of inventory on demand is small under a large backorder volume so demand does not increase to infinity. Therefore, the firm cannot induce arbitrarily high demand by creating an arbitrarily long wait list. The underlying intuition of the boundedness of $\gamma(\cdot)$ is that the high demand induced by a long wait list is canceled out by the impatience it arouses.

ASSUMPTION 3. *Let*

$$R(d_t, I_t^a) := (p(d_t) - b - \alpha(c + r_d))(d_t + \gamma(I_t^a)). \quad (2)$$

$R(d_t, I_t^a)$ *is jointly concave in (d_t, I_t^a) on its domain.*

Assumption 3 is imposed mainly for technical tractability because it is required to establish the joint concavity of the objective and value functions in each period (see the discussions after Lemma 4). Note that $R(d_t, I_t^a)$ is the expected difference between the revenue and the total cost (i.e., procuring, displaying, and backlogging costs) to satisfy the current demand in the next period when the firm holds a customer-accessible inventory I_t^a and charges a sales price $p(d_t)$. The joint concavity of $R(\cdot, \cdot)$ implies that the expected difference between the revenue and the total cost to meet the current demand in the next period has decreasing marginal values with respect to both the expected price-induced demand and customer-accessible inventory level. The joint concavity of $R(\cdot, \cdot)$ is stronger than the concavity of expected revenue (Assumption 1) because it also captures the impact of inventory-dependent demand on revenue and procurement, reallocation, and backlogging costs. We discuss this assumption in detail in the following subsection.

3.1. Discussions on Assumption 3

Assumption 3 is essential to show the analytical results in this paper. We first characterize the necessary and sufficient condition for Assumption 3:

LEMMA 1. *$R(d_t, I_t^a)$ is jointly concave in (d_t, I_t^a) on its domain if and only if*

$$(p''(d_t)(d_t + \gamma(I_t^a)) + 2p'(d_t))(p(d_t) - b - \alpha(c + r_d))\gamma''(I_t^a)$$

$$\geqslant (p'(d_t)\gamma'(I_t^a))^2, \quad for\ all\ d_t \in [\underline{d}, \bar{d}]\ and\ I_t^a \leqslant K_a. \quad (3)$$

Condition (3) is complicated and somewhat difficult to understand. Hence, we give the following simpler necessary condition for Assumption 3 to hold:

LEMMA 2. *If $R(\cdot, \cdot)$ is jointly concave on its domain, then we have:*

(a) *For any I_t^a such that $\gamma''(I_t^a) = 0$, $\gamma'(I_t^a) = 0$ as well. Therefore, there exists a threshold $I^* \leqslant K_a$ (I^* may be $-\infty$), such that*

$$\gamma'(I_t^a) \begin{cases} < 0, & if\ I_t^a > I^*, \\ = 0, & otherwise, \end{cases}$$

and

$$\gamma''(I_t^a) \begin{cases} < 0, & if\ I_t^a > I^*, \\ = 0, & otherwise. \end{cases}$$

(b) *There exists an $0 < M < +\infty$, such that, for any $I_t^a \leqslant K_a$, $(\gamma'(I_t^a))^2 \leqslant -M\gamma''(I_t^a)$.*

Lemma 2(a) shows that, if Assumption 3 is satisfied, there exists a threshold inventory level I^*, such that there is no scarcity effect for all customer-accessible inventory level below this threshold and the scarcity function is strictly decreasing and strictly concave for all customer-accessible inventory level above this threshold. Lemma 2(b) proves that $R(\cdot, \cdot)$ is jointly concave only if, for all I_t^a, compared with $|\gamma'(I_t^a)|$, $|\gamma''(I_t^a)|$ is sufficiently big. In other words, in the region where the scarcity effect exists (i.e., $\gamma'(I_t^a) < 0$), the curvature of the function $\gamma(\cdot)$ should be sufficiently big. This condition is not restrictive and, for example, can be satisfied by the commonly used power or exponential families of *scarcity functions*. We note that, mathematically, Lemma 2(a) is a corollary of Lemma 2(b). Next, we show that the necessary condition characterized in Lemma 2(b) is also sufficient to some extent.

LEMMA 3. *If there exists an $0 < M < +\infty$, such that, for any $I_t^a \leqslant K_a$, $(\gamma'(I_t^a))^2 \leqslant -M\gamma''(I_t^a)$, the following statements hold:*

(a) *For any inverse demand curve $p(\cdot)$, there exists a threshold $\delta^* < +\infty$, such that, for any $\delta \geqslant \delta^*$, with $\hat{p}_\delta(\cdot) := p(\cdot) + \delta$,*

$$\hat{R}_\delta(d_t, I_t^a) := (\hat{p}_\delta(d_t) - b - \alpha(c + r_d))(d_t + \gamma(I_t^a))$$

is jointly concave in (d_t, I_t^a) for $d_t \in [\underline{d}, \bar{d}]$ and $I_t^a \leqslant K_a$.

(b) *Suppose that $p''(\cdot) \neq 0$ for any $d_t \in [\underline{d}, \bar{d}]$. For any scarcity function $\gamma(\cdot)$, there exists a threshold $\varsigma^* < +\infty$, such that, for any $\varsigma \geqslant \varsigma^*$, with $\hat{\gamma}_\varsigma(\cdot) := \gamma(\cdot) + \varsigma$, $\hat{R}_\varsigma(d_t, I_t^a) := (p(d_t) - b - \alpha(c + r_d))(d_t + \hat{\gamma}_\varsigma(I_t^a))$ is jointly concave in (d_t, I_t^a) for $d_t \in [\underline{d}, \bar{d}]$ and $I_t^a \leqslant K_a$.*

Lemma 3 demonstrates that, as long as the condition characterized in Lemma 2(b) on the scarcity functions, $\gamma(\cdot)$, is satisfied, $R(\cdot, \cdot)$ is jointly concave on its domain

if (a) the sales price of the product, $p(\cdot)$, is sufficiently high relative to the inverse of price sensitivity, $|p'(\cdot)|$; or (b) the price is not linear in demand, and the scarcity effect driven demand, $\gamma(\cdot)$, is sufficiently high relative to the *scarcity intensity*, $|\gamma'(\cdot)|$. These sufficient conditions have a clear economic interpretation: The price elasticity of demand (i.e., $|(d\,d_t/d_t)/(d\,p_t/p_t)|$) is sufficiently high relative to the *inventory elasticity of demand* (defined as $|(d\gamma/\gamma)/(dI_t^a/I_t^a)|$). In practice, this condition is not restrictive. Compared with the primary demand leverage (i.e., the sales price), the customer-accessible inventory (through the scarcity effect) has less impact on the potential demand because not every customer cares about the backlogging risk of a product, but everyone cares about its price. Therefore, Assumption 3 can be satisfied under a mild condition with economic interpretation.

In the online appendix (Section EC.2), we discuss the case where the inverse demand function $p(\cdot)$ and the scarcity function $\gamma(\cdot)$ belong to the power and/or exponential function families, which are the most commonly used inverse demand functions and scarcity functions in the literature (see, e.g., Sapra et al. 2010). For this case, we characterize necessary and sufficient conditions for Assumption 3 in model primitives, which are easy to verify.

Finally, when Assumption 3 does not hold (i.e., $R(\cdot, \cdot)$ is not jointly concave), we have conducted extensive numerical experiments to test the robustness of our analytical results. Our numerical results verify that the analytical characterizations of the optimal policies in our model are robust and hold for nonconcave $R(\cdot, \cdot)$'s in all of our experiments. In particular, Lemma 2 implies that when the scarcity function $\gamma(\cdot)$ contains a linear and strictly decreasing piece, $R(\cdot, \cdot)$ is not jointly concave. We present our numerical experiments for this case in §7.1.

4. Unified Model

In this section, we propose a unified model to analyze the joint pricing and inventory replenishment/disposal/reallocation problem when the firm faces random demand that is negatively correlated with the customer-accessible leftover inventory. We characterize the structure of the optimal pricing and inventory policy and give sufficient conditions under which the firm does not (a) dispose of its on-hand inventory, (b) withhold any inventory, (c) reallocate its customer-accessible inventory to the warehouse, or (d) display any positive inventory to customers.

This model is suitable for the case wherein the firm can both withhold its on-hand inventory in its private warehouse not observable by customers (e.g., clothing and electronics markets) and dispose of it (e.g., in the hi-tech industry, the evolution of product generation is so fast that the retailers/manufacturers have to sell excess old versions at a significantly discounted price). When potential demand is negatively correlated with the customer-accessible leftover inventory, the firm faces greater overage risk because a

high customer-accessible leftover inventory not only incurs a high holding cost but also suppresses potential demand. Both inventory withholding and inventory disposal policies enable the firm to strategically maintain a low customer-accessible inventory, so as to induce high potential demand and mitigate the overstocking risk. Hence, we incorporate inventory withholding and inventory disposal into our unified model.

The unified model is quite general and can be reduced to several specific models that are of interest on their own. For example, we show that if the warehouse holding cost h_w is sufficiently large, the unified model is reduced to the one without inventory withholding, which is discussed in detail in §5.1. Besides, if the disposal salvage value s is sufficiently low, the unified model is reduced to the one without inventory disposal, which is discussed in detail in §5.2.

To formulate the planning problem as a dynamic program, let:

$V_t(I_t^a, I_t) =$ the maximum expected discounted profits in periods $t, t-1, \ldots, 1$, when starting period t with a customer-accessible inventory level I_t^a and a total inventory level I_t.

Without loss of generality, we assume that the excess inventory in the last period (period 1) is discarded without any salvage value, i.e., $V_0(I_0^a, I_0) = 0$, for any (I_0^a, I_0).

The optimal value functions satisfy the following recursive scheme:

$$V_t(I_t^a, I_t) = r_d I_t^a + c I_t + \max_{(x_t^a, x_t, d_t) \in F(I_t^a)} J_t(x_t^a, x_t, d_t, I_t^a, I_t), \quad (4)$$

where $F(I_t^a) := \{(x_t^a, x_t, d_t): x_t^a \in [\min\{I_t^a, 0\}, K_a], x_t \geqslant x_t^a, d_t \in [\underline{d}, \bar{d}]\}$ denotes the set of feasible inventory and pricing decisions, and

$$
\begin{aligned}
&J_t(x_t^a, x_t, d_t, I_t^a, I_t) \\
&= -r_d I_t^a - c I_t + p(d_t)\mathbb{E}[\delta(p(d_t), I_t^a, \epsilon_t)] - c(x_t - I_t)^+ \\
&\quad + s(x_t - I_t)^- - r_d(x_t^a - I_t^a)^+ - r_w(x_t^a - I_t^a)^- - h_w(x_t - x_t^a) \\
&\quad - \mathbb{E}\{h_a(x_t^a - \delta(p(d_t), I_t^a, \epsilon_t))^+ + b(x_t^a - \delta(p(d_t), I_t^a, \epsilon_t))^-\} \\
&\quad + \alpha\mathbb{E}\{V_{t-1}(x_t^a - \delta(p(d_t), I_t^a, \epsilon_t), x_t - \delta(p(d_t), I_t^a, \epsilon_t))\} \\
&= p(d_t)(d_t + \gamma(I_t^a)) - (c-s)(x_t - I_t)^- - (h_w + c)x_t \\
&\quad - (r_d + r_w)(x_t^a - I_t^a)^- + (h_w - r_d)x_t^a \\
&\quad + \mathbb{E}\big[(b + \alpha r_d)(x_t^a - (d_t + \gamma(I_t^a))\epsilon_t^m - \epsilon_t^a) \\
&\quad + \alpha c(x_t - (d_t + \gamma(I_t^a))\epsilon_t^m - \epsilon_t^a)\big] \\
&\quad + \mathbb{E}\big\{\alpha[V_{t-1}(x_t^a - (d_t + \gamma(I_t^a))\epsilon_t^m - \epsilon_t^a, x_t \\
&\quad - (d_t + \gamma(I_t^a))\epsilon_t^m - \epsilon_t^a) - r_d(x_t^a - (d_t + \gamma(I_t^a))\epsilon_t^m - \epsilon_t^a) \\
&\quad - c(x_t - (d_t + \gamma(I_t^a))\epsilon_t^m - \epsilon_t^a)] \\
&\quad - (b + h_a)(x_t^a - (d_t + \gamma(I_t^a))\epsilon_t^m - \epsilon_t^a)^+\big\}
\end{aligned}
$$

$$= (p(d_t) - \alpha(c + r_d) - b)(d_t + \gamma(I_t^a)) - (c - s)(x_t - I_t)^-$$
$$- (r_d + r_w)(x_t^a - I_t^a)^- - (h_w + (1 - \alpha)c)x_t$$
$$+ (h_w + b - (1 - \alpha)r_d)x_t^a$$
$$+ \mathbb{E}\{-(h_a + b)(x_t^a - (d_t + \gamma(I_t^a))\epsilon_t^m - \epsilon_t^a)^+$$
$$+ \alpha[V_{t-1}(x_t^a - (d_t + \gamma(I_t^a))\epsilon_t^m - \epsilon_t^a, x_t$$
$$- (d_t + \gamma(I_t^a))\epsilon_t^m - \epsilon_t^a) - r_d(x_t^a - (d_t + \gamma(I_t^a))\epsilon_t^m - \epsilon_t^a)$$
$$- c(x_t - (d_t + \gamma(I_t^a))\epsilon_t^m - \epsilon_t^a)]\}$$
$$= R(d_t, I_t^a) - \theta(x_t - I_t)^- - (r_d + r_w)(x_t^a - I_t^a)^- - \psi x_t + \phi x_t^a$$
$$+ \mathbb{E}\{G_t(x_t^a - (d_t + \gamma(I_t^a))\epsilon_t^m - \epsilon_t^a, x_t$$
$$- (d_t + \gamma(I_t^a))\epsilon_t^m - \epsilon_t^a)\}, \tag{5}$$

where $G_t(x, y) := -(b + h_a)x^+ + \alpha(V_{t-1}(x, y) - r_d x - cy)$,

$$\theta := c - s = \text{the unit loss of inventory} \atop \text{disposal}, \tag{6}$$

$$\psi := h_w + (1 - \alpha)c = \text{the unit cost of} \atop \text{replenishing and holding inventory} \atop \text{in the warehouse,}$$

$$\phi := h_w + b - (1 - \alpha)r_d$$

$$= \text{the unit saving of reallocating} \atop \text{warehouse inventory to the} \atop \text{customer-accessible storage.}$$

We use $(x_t^{a*}(I_t^a, I_t), x_t^*(I_t^a, I_t), d_t^*(I_t^a, I_t))$ to denote the maximizer in (4), which stands for the optimal policy in period t, with customer-accessible inventory level I_t^a and total inventory level I_t. To characterize the structure of the optimal inventory replenishment/disposal/reallocation and pricing policies, we define the following optimizers: $(x_t^a(I_t^a), d_t(I_t^a))$ and $(\tilde{x}_t^a(I_t^a), \tilde{x}_t(I_t^a), \tilde{d}_t(I_t^a))$. Let

$$(x_t^a(I_t^a), d_t(I_t^a)) := \mathop{\arg\max}_{x_t^a \in [\min\{I_t^a, 0\}, K_a], d_t \in [\underline{d}, \bar{d}]} \{R(d_t, I_t^a) + \beta x_t^a$$

$$+ \mathbb{E}[G_t(x_t^a - \delta(p(d_t), I_t^a, \epsilon_t), x_t^a$$
$$- \delta(p(d_t), I_t^a, \epsilon_t))]\}, \tag{7}$$

where $\beta := b - (1 - \alpha)(c + r_d) > 0$. \tag{8}

Note that $x_t^a(I_t^a)$ is the optimal order-up-to inventory level, if the firm procures positive inventory and displays all of its on-hand inventory to customers, whereas $d_t(I_t^a)$ is the optimal expected price-induced demand in this case. Let

$$(\tilde{x}_t^a(I_t^a), \tilde{x}_t(I_t^a), \tilde{d}_t(I_t^a))$$

$$:= \mathop{\arg\max}_{(x_t^a, x_t, d_t) \in F(I_t^a)} \{R(d_t, I_t^a) + (\theta - \psi)x_t$$

$$- (r_d + r_w)(x_t^a - I_t^a)^- + \phi x_t^a$$
$$+ \mathbb{E}\{G_t(x_t^a - \delta(p(d_t), I_t^a, \epsilon_t), x_t - \delta(p(d_t), I_t^a, \epsilon_t))\}\}. \tag{9}$$

When the firm disposes of its on-hand inventory, $\tilde{x}_t^a(I_t^a)$ is the optimal display-up-to inventory level and $\tilde{x}_t(I_t^a)$ is the optimal dispose-down-to inventory level, whereas $\tilde{d}_t(I_t^a)$ is the optimal expected price-induced demand. The following lemma establishes the properties of the two optimizers:

LEMMA 4. *For each $t = T, T - 1, \ldots, 1$, the following statements hold*:

(a) $J_t(x_t^a, x_t, d_t, I_t^a, I_t)$ *is jointly concave and continuously differentiable in* $(x_t^a, x_t, d_t, I_t^a, I_t)$ *except for a set of measure zero; for any fixed* (I_t^a, I_t), $J_t(\cdot, \cdot, \cdot, I_t^a, I_t)$ *is strictly jointly concave in* (x_t^a, x_t, d_t).

(b) $V_t(I_t^a, I_t)$ *is jointly concave and continuously differentiable in* (I_t^a, I_t), *whereas* $V_t(I_t^a, I_t) - r_d I_t^a - c I_t$ *is decreasing in I_t^a and I_t.*

Lemma 4 proves that the objective function in each period is jointly concave and almost everywhere differentiable and the value function is jointly concave and continuously differentiable. Moreover, the second half of Lemma 4(b) implies that the normalized value function, $V_t(I_t^a, I_t) - r_d I_t^a - c I_t$, is decreasing in both the customer-accessible inventory level I_t^a and the total inventory level I_t, which generalizes Proposition 5.1 in Sapra et al. (2010). We note that the joint concavity of $R(\cdot, \cdot)$ on its entire domain is necessary to prove that the objective functions $J_t(\cdot, \cdot, \cdot, I_t^a, I_t)$ and that the value functions $V_t(\cdot, \cdot)$ are jointly concave, which is essential to analytically establish other structural results in our paper. We can easily find examples in which $R(\cdot, \cdot)$ fails to be jointly concave (e.g., $\gamma(\cdot)$ contains a linear and strictly decreasing piece) and leads to nonconcave $J_t(\cdot, \cdot, \cdot, I_t^a, I_t)$s and $V_t(\cdot, \cdot)$s. In this case, we are unable to analytically show the structural results in our paper (e.g., Theorem 1 and Theorem 3). In §7.1, we numerically test whether the structure of the optimal policy characterized in our theoretical model still holds. With the help of Lemma 4, we characterize the structural properties of the optimal policy in the unified model as follows:

THEOREM 1. *For $t = T, T - 1, \ldots, 1$, the following statements hold*:

(a) $x_t^a(I_t^a) \leqslant \tilde{x}_t(I_t^a)$. *Moreover, let*

$$q_t^*(I_t^a, I_t) := x_t^*(I_t^a, I_t) - I_t$$

denote the optimal order/disposal quantity and we have:

$$q_t^*(I_t^a, I_t) \begin{cases} > 0 & \text{if } I_t < x_t^a(I_t^a), \\ = 0 & \text{if } x_t^a(I_t^a) \leqslant I_t \leqslant \tilde{x}_t(I_t^a), \\ < 0 & \text{otherwise}, \end{cases}$$

i.e., it is optimal to order if and only if $I_t < x_t^a(I_t^a)$ and to dispose if and only if $I_t > \tilde{x}_t(I_t^a)$.

(b) *If $I_t < x_t^a(I_t^a)$,*

$$x_t^{a*}(I_t^a, I_t) = x_t^*(I_t^a, I_t) = x_t^a(I_t^a), \quad d_t^*(I_t^a, I_t) = d_t(I_t^a),$$

i.e., it is optimal to order and display up to $x_t^a(I_t^a)$ and charge a list price $p(d_t(I_t^a))$.

(c) *If $I_t > \tilde{x}_t(I_t^a)$,*

$$(x_t^{a*}(I_t^a, I_t), x_t^*(I_t^a, I_t), d_t^*(I_t^a, I_t)) = (\tilde{x}_t^a(I_t^a), \tilde{x}_t(I_t^a), \tilde{d}_t(I_t^a)),$$

i.e., it is optimal to dispose the total inventory level down to $\tilde{x}_t(I_t^a)$, display $\tilde{x}_t^a(I_t^a)$, and charge a list price $p(\tilde{d}_t(I_t^a))$).
  (d) *If $I_t \in [x_t^a(I_t^a), \tilde{x}_t(I_t^a)]$, $x_t^*(I_t^a, I_t) = I_t$, i.e., it is optimal to keep the total inventory level.*
  (e) *$x_t^a(I_t^a)$ is continuously decreasing in I_t^a, whereas $d_t(I_t^a)$ is continuously increasing in I_t^a.*

Theorem 1 generalizes Proposition 5 in Sapra et al. (2010) by characterizing the structure of the optimal policy in our unified model. We show that a customer-accessible inventory-dependent order-up-to/dispose-down-to/display-up-to list-price policy is optimal. The optimal policy is characterized by two thresholds, i.e., the ordering threshold $x_t^a(I_t^a)$ and the disposal threshold $\tilde{x}_t(I_t^a)$, both of which depend on the customer-accessible inventory level, I_t^a. If the total inventory level, I_t, is below the ordering threshold, i.e., $I_t < x_t^a(I_t^a)$, the firm should order-up-to this threshold, display all of its on-hand inventory to customers, and charge a customer-accessible inventory-dependent list-price $p(d_t(I_t^a))$. If the total inventory level is higher than the disposal threshold, i.e., $I_t > \tilde{x}_t(I_t^a)$, the firm should dispose-down-to this threshold, display part of its on-hand inventory, $\tilde{x}_t^a(I_t^a)$, to customers, and charge a customer-accessible inventory-dependent list-price $p(\tilde{d}_t(I_t^a))$. If the total inventory level is between the above two thresholds, i.e., $I_t \in [x_t^a(I_t^a), \tilde{x}_t(I_t^a)]$, the firm should keep its total net inventory and display part of it to customers. In particular, Theorem 1(b) implies that if it is optimal to order, the firm should not withhold anything. Order-and-withhold policy is dominated by displaying the same amount of inventory to customers but not ordering the inventory that will be withheld (so no inventory will be withheld). This is intuitive, because the marginal cost of order-and-withhold is at least $c + h_w$ (procurement cost and holding cost in the warehouse), while the marginal benefit of inventory withholding is at most αc (saving from the purchasing cost in the next period). Moreover, part (e) of Theorem 1 demonstrates that as the excess customer-accessible inventory level increases, lower demand is induced and the firm has a greater incentive to turn it over, both of which give rise to lower optimal order-up-to levels and optimal sales prices.

The firm's excess inventory generally has three impacts on the performance of the system: (1) satisfying future demand, (2) incurring holding costs, and (3) inducing/suppressing potential demand, the first with positive marginal value and the other two with negative marginal values. Hence, after normalizing the first effect ($V_t(I_t^a, I_t) - r_d I_t^a - c I_t$), the value-to-go function of the firm is decreasing in its customer-accessible inventory level and total inventory level. To better address the intertwined trade-off between these three effects, the firm can adopt dynamic pricing, inventory withholding, and inventory disposal

strategies. As suggested in Theorem 1, the firm needs to price the product in accordance with the customer-accessible inventory level so as to better control the scarcity effect of demand. Theorem 1 also shows that when the total inventory is high, the firm should withhold and dispose of its on-hand inventory, which saves holding costs and mitigates the risk of suppressing potential demand. On the other hand, the opportunity to redisplay the withheld inventory in the warehouse enables the firm to satisfy potential demand without discouraging it. In short, combining dynamic pricing, inventory withholding, and inventory disposal policies helps the firm better match supply and demand and greatly enhances its profitability.

We now analyze how the model primitives influence the firm's optimal operational decisions, such as inventory disposal, inventory withholding, and inventory display.

THEOREM 2. *The following statements hold:*
  (a) *If $h_w \geqslant \alpha c - s$, $\tilde{x}_t(I_t^a) = \tilde{x}_t^a(I_t^a)$ for any $t = T, T-1, \ldots, 1$.*
  (b) *There exists an $s_* < c$, such that, if $s \leqslant s_*$, $\tilde{x}_t(I_t^a) = +\infty$ for any $I_t^a \leqslant K_a$ and $t = T, T-1, \ldots, 1$.*
  (c) *If $\inf_{I_t^a < K_a} \gamma'(I_t^a) \geqslant -M$, for some $M < +\infty$, there exists an $r_* < +\infty$, such that, if $r_w \geqslant r_*$, $\tilde{x}_t^a(I_t^a) \geqslant I_t^a$, for any $I_t^a \leqslant K_a$ and $t = T, T-1, \ldots, 1$. On the other hand, if $\inf_{I_t^a < K_a} \gamma'(I_t^a) = -\infty$, for any $r_w > 0$, there exists a threshold $I_t^*(r_w) < K_a$, such that, if $I_t^a \geqslant I_t^*(r_w)$, $\tilde{x}_t^a(I_t^a) < I_t^a$, for any $t = T, T-1, \ldots, 2$.*
  (d) *Let $\iota < 1$, and $\bar{D} := \sup\{\Delta: P(D_t \geqslant \Delta) \geqslant \iota\}$, i.e., the probability that the demand in period t exceeds $\bar{D}$ is smaller than ι, regardless of the policy the firm employs. If*

$$\alpha(\underline{p} - b - \alpha(c + r_d) + \underline{m}\beta)(1-\iota)\gamma'(-\bar{D})$$
$$+ (r_d + r_w + \phi) \leqslant 0, \tag{10}$$

then $x_t^{a*}(I_t^a, I_t) \leqslant 0$ for any $I_t^a \leqslant K_a$, I_t, and $t = T, T-1, \ldots, 1$.

Theorem 2(a) shows that, when the warehouse holding cost is sufficiently high ($h_w \geqslant \alpha c - s$), the firm should display all of its on-hand inventory to customers. Part (b) demonstrates that, when inventory disposal is sufficiently costly ($s \leqslant s_*$), the firm would rather not dispose any of its inventory, regardless of its total inventory level. When the condition in part (a) [part (b)] holds, the unified model is reduced to the model without inventory withholding [inventory disposal]. This generates additional insights and is thoroughly discussed in §5.1 [§5.2]. Theorem 2(c) reveals that the optimal inventory reallocation balances the trade-off between saving the current reallocation cost and stimulating future demand. More specifically, if the intensity of scarcity effect is bounded, the firm should not reallocate its inventory from the customer-accessible storage to the warehouse, as long as the reallocation fee is sufficiently high. Otherwise (i.e., the intensity of scarcity effect is unbounded), the firm should always withhold part of

its inventory in the warehouse, if the excess customer-accessible inventory level is high enough.

Theorem 2(d) shows that when the demand-stimulating effect/scarcity effect of inventory is sufficiently strong (characterized by (10)), the backlogging cost incurred by the wait list is dominated by the revenue generated by the scarcity effect. Therefore, the firm should not display any positive inventory, and every customer is placed on a wait list before receiving the product. This analytical result justifies the marketing strategy adopted by, e.g., BMW, in which the availability of the Mini Cooper is intentionally limited and more customers are attracted by its wait list.

5. Additional Results in Two Special Cases

In this section, we study two special cases of our unified model that are of interest on their own, i.e., the model without inventory withholding and the model without inventory disposal. As shown in Theorem 2, when it is too expensive to withhold [dispose] inventory, it is optimal for the firm not to withhold [dispose of] any inventory. These two special cases deliver new results and sharper insights on the impact of the inventory-dependent demand on the firm's pricing and inventory decisions. We also characterize how the operational flexibilities (e.g., an increase in the salvage value and the inventory withholding opportunity) help the firm to mitigate the additional overage risk caused by inventory-dependent demand.

5.1. Without Inventory Withholding

In some circumstances, the firm cannot store its inventory in the warehouse. Such storage may be too costly or transportation too inconvenient. For instance, car dealers usually display all of their automobiles because withholding and redisplaying the inventory is too costly and inconvenient. In this subsection, we confine our analysis to the model without inventory withholding. In this model, because no inventory is stored in the warehouse, the state space dimension is reduced to one. This reduction offers new results and sharper insights into how the inventory-dependent demand influences the firm's optimal decisions. More specifically, we demonstrate that the scarcity effect of inventory increases the overstocking risk and, thus, drives the firm to set a lower order-up-to level and charge a lower sales price. On the other hand, when the firm has higher disposal flexibility (i.e., a higher salvage value), it can more easily mitigate such overage risk by disposing of its surplus inventory. We show that the firm with a higher salvage value sets higher order-up-to levels and sales prices.

To formulate the planning problem as a dynamic program, let:

$V_t^s(I_t^a) =$ the maximum expected discounted profits in periods $t, t-1, \ldots, 1$, when starting period t with a customer-accessible inventory level I_t^a.

Because no inventory is withheld in the warehouse in this model, $I_t^a = I_t$, there is no need to record the total inventory level I_t. Therefore, the state space dimension is reduced to one. Similarly, we will not incur the warehouse inventory holding cost (h_w), the redisplay cost (r_d), and the withholding cost (r_w) in this model. The superscript "s" refers to "single location storage."

Without loss of generality, we assume that the excess inventory in the last period (period 1) is discarded without any salvage value, i.e., $V_0^s(I_0^a) = 0$, for any $I_0^a \leqslant K_a$. The value functions satisfy the following recursive scheme:

$$V_t^s(I_t^a) = cI_t^a + \max_{(x_t^a, d_t) \in F^s(I_t^a)} J_t^s(x_t^a, d_t, I_t^a),$$

where $F^s(I_t^a) := [\min\{0, I_t^a\}, K_a] \times [\underline{d}, \bar{d}]$ denotes the set of feasible order-up-to/dispose-down-to levels and expected price-induced demand, and

$$J_t^s(x_t^a, d_t, I_t^a)$$
$$= p(d_t)\mathbb{E}[\delta(p(d_t), I_t^a, \epsilon_t)] + s(x_t^a - I_t^a)^- - c(x_t^a - I_t^a)^+$$
$$\quad - cI_t^a - \mathbb{E}[b(x_t^a - \delta(p(d_t), I_t^a, \epsilon_t))^-$$
$$\quad + h_a(x_t^a - \delta(p(d_t), I_t^a, \epsilon_t))^+]$$
$$\quad + \alpha\mathbb{E}[V_{t-1}^s(x_t^a - \delta(p(d_t), I_t^a, \epsilon_t))].$$

Following the algebraic manipulation similar to that used in (5), we obtain:

$$J_t^s(x_t^a, d_t, I_t^a) = R^s(d_t, I_t^a) + \beta^s x_t^a - \theta(x_t^a - I_t^a)^-$$
$$\quad + \mathbb{E}[G_t^s(x_t^a - \delta(p(d_t), I_t^a, \epsilon_t))],$$

where $R^s(d_t, I_t^a) := (p(d_t) - b - \alpha c)(d_t + \gamma(I_t^a))$,
$$G_t^s(y) := -(b + h_a)y^+ + \alpha[V_{t-1}^s(y) - cy],$$
$$\beta^s := b - (1 - \alpha)c, \quad (11)$$

and θ is defined in (6). Note that, under Assumption 3, $R^s(d_t, I_t^a) = R(d_t, I_t^a) + \alpha r_d(d_t + \gamma(I_t^a))$ is jointly concave on its domain.

As a corollary to Theorem 1, the optimal policy in the model without inventory withholding is an inventory-dependent order-up-to/dispose-down-to list-price policy, as shown below:

THEOREM 3. *Consider a model without inventory withholding. For each* $t = T, T-1, \ldots, 1$, *the following statements hold*:

(a) $g_t^s(x_t^a, d_t, I_t^a) := \mathbb{E}[G_t^s(x_t^a - \delta(p(d_t), I_t^a, \epsilon_t))]$ *is jointly concave and continuously differentiable in* (x_t^a, d_t, I_t^a) *if* $x_t^a \neq I_t^a$; *for any fixed* I_t^a, $g_t^s(\cdot, \cdot, I_t^a)$ *is strictly concave.*

(b) $V_t^s(I_t^a)$ *is concave in* I_t^a. $V_t^s(I_t^a) - cI_t^a$ *is decreasing and continuously differentiable in* I_t^a.

(c) $J_t^s(\cdot, \cdot, I_t^a)$ *is strictly concave for any fixed* I_t^a, *and there exists a unique* $(x_t^{s*}(I_t^a), d_t^{s*}(I_t^a))$ *such that*

$$(x_t^{s*}(I_t^a), d_t^{s*}(I_t^a)) = \underset{(x_t, d_t) \in F^s(I_t^a)}{\arg\max} J_t^s(x_t^a, d_t, I_t^a).$$

(d) *Let* $q_t^{s*}(I_t^a) = x_t^{s*}(I_t^a) - I_t^a$ *denote the optimal order/disposal quantity. There exist two threshold inventory levels* I_t^H *and* I_t^L *(* $I_t^L < I_t^H$ *), such that,*

$$q_t^{s*}(I_t^a) \begin{cases} > 0 & \text{if } I_t^a < I_t^L, \\ = 0 & \text{if } I_t^L \leqslant I_t^a \leqslant I_t^H, \\ < 0 & \text{otherwise,} \end{cases}$$

i.e., the firm should order if its inventory level I_t^a *is less than the lower threshold* I_t^L*, dispose if it is more than the higher threshold* I_t^H*, and not order or dispose if it is between the two thresholds.*

(e) *If* $I_t^a < I_t^L$ *or* $I_t^a > I_t^H$*, the optimal order-up-to/dispose-down-to level* $x_t^{s*}(I_t^a)$ *is decreasing in* I_t^a*. If* $I_t^L \leqslant I_t^a \leqslant I_t^H$*, the optimal inventory after replenishment/disposal is increasing in* I_t^a*.*

(f) *The optimal price-induced-demand* $d_t^{s*}(I_t^a)$ *is increasing in* I_t^a*.*

Theorem 3 implies that, when the firm cannot withhold its on-hand inventory, the optimal policy is to order when the customer-accessible inventory level is low (below I_t^L), to dispose of it when it is high (above I_t^H), and to make no adjustments when it is between the two thresholds. The optimal order-up-to/dispose-down-to and list-price levels are customer-accessible-inventory-dependent. As shown in Theorem 3, when the customer-accessible inventory level is higher, order-up-to/dispose-down-to levels and sales prices are lower because a high customer-accessible inventory level suppresses potential demand and the firm has a strong incentive to turn it over.

We now analyze how the scarcity effect of inventory impacts optimal pricing and inventory policies. Compared with the model in which demand is independent of inventory, when potential demand is negatively correlated with customer-accessible leftover inventory levels, the marginal value of on-hand inventory decreases and the firm suffers from the demand reduction caused by a high inventory level. As a result, the firm should order less/dispose of more to mitigate the additional overstocking risk caused by the scarcity effect of inventory. At the same time, to better catch the sales opportunity, it is optimal to underprice the product so as to attract more customers. Moreover, in a market where the firm has little power to set the sales price, we show a sharper result: With a more intensive scarcity effect, the firm should keep a lower inventory level after replenishment/disposal. The following theorem formalizes these intuitions.

Theorem 4. *Consider a model without inventory withholding. Assume that* $D_t = \delta(d_t, I_t^a, \epsilon_t)$ *and* $\hat{D}_t = \hat{\delta}(d_t, I_t^a, \epsilon_t)$ *with inventory dependent term* $\gamma(I_t^a)$ *and* $\hat{\gamma}(I_t^a)$*, respectively. We also assume that the demand is of additive form (i.e.,* $\epsilon_t^m = 1$ *with probability* 1*). The following statements hold:*

(a) *Assume that* $\hat{\gamma}(I_t^a) = \gamma_0 = \lim_{x \to -\infty} \gamma(x)$ *for all* $I_t^a \leqslant K_a$*, i.e.,* $\hat{D}_t$ *does not depend on the customer-accessible inventory level. We show that* $I_t^L \leqslant \hat{I}_t^L$*,* $I_t^H \leqslant \hat{I}_t^H$*,* $x_t^{s*}(I_t^a) \leqslant \hat{x}_t^{s*}(I_t^a)$ *and* $d_t^{s*}(I_t^a) \geqslant \hat{d}_t^{s*}(I_t^a)$ *for all* $I_t^a \leqslant K_a$*.*

(b) *Assume that* $\gamma'(I_t^a) \leqslant \hat{\gamma}'(I_t^a)$ *for all* $I_t^a \leqslant K_a$ *and that* $\lim_{I_t^a \to -\infty} \gamma(I_t^a) = \lim_{I_t^a \to -\infty} \hat{\gamma}(I_t^a) = \gamma_0$*. Let* $\underline{p} = \bar{p} = p_0$ *and* $d_0 = d(p_0)$*. We have* $I_t^L \leqslant \hat{I}_t^L$*,* $I_t^H \leqslant \hat{I}_t^H$ *and* $x_t^{s*}(I_t^a) \leqslant \hat{x}_t^{s*}(I_t^a)$ *for all* $I_t^a \leqslant K_a$*.*

As a generalization of Theorem 3.2 in Sapra et al. (2010) to the model with dynamic pricing and inventory disposal, Theorem 4 shows that the firm should understock and underprice the product under the scarcity effect of inventory. In Theorem 4, we need the additive demand assumption, i.e., $\epsilon_t^m = 1$ almost surely. The additive demand model is widely applied in the joint pricing and inventory control literature (see, e.g., Li and Zheng 2006, Feng 2010 and Pang et al. 2012), primarily because it enhances technical tractability and facilitates analysis. To show Theorem 4 and other comparisons between the optimizers in different models (Theorems 5–7 below), we need to iteratively establish comparisons between the derivatives of value functions. The additive demand form is necessary to link the monotonicity relationship between optimizers and that between derivatives. All results in this paper, except Theorems 4–7, hold for the more general demand form introduced in (1).

Efficiently disposing of surplus inventory protects the firm from the demand-suppressing effect of inventory. As the salvage value increases, the cost of inventory disposal decreases, and the firm has greater disposal flexibility. We characterize the impact of the salvage value on the optimal pricing and inventory decisions in the following theorem:

Theorem 5. *Consider a model without inventory withholding. For any* $t = T, T-1, \dots, 1$*, assume that the demand is of additive form (i.e.,* $\epsilon_t^m = 1$ *with probability* 1*), and* $s < \hat{s}$*.*

(a) $\partial_{I_t^a} \hat{V}_t^s(I_t^a) \geqslant \partial_{I_t^a} V_t^s(I_t^a)$*.*

(b) $\hat{I}_t^L \geqslant I_t^L$*.*

(c) $\hat{x}_t^{s*}(I_t^a) \geqslant x_t^{s*}(I_t^a)$ *and, hence,* $\hat{q}_t^{s*}(I_t^a) \geqslant q_t^{s*}(I_t^a)$ *for all* $I_t^a \leqslant \hat{I}_t^H$*.*

(d) $\hat{d}_t^{s*}(I_t^a) \leqslant d_t^{s*}(I_t^a)$*.*

Theorem 5(a) shows that the marginal value of on-hand inventory increases in the salvage value. Parts (b)–(d) demonstrate that with a higher salvage value, the firm should set higher ordering thresholds, order-up-to levels, and sales prices. Recall from Theorem 4 that the inventory-dependent demand strengthens overstocking risk by suppressing potential demand so that optimal order-up-to/disposal-down-to levels and optimal sales prices are lower in the model with inventory-dependent demand than those in the model with inventory-independent demand. On the other hand, Theorem 5 demonstrates that increased operational flexibility (i.e., a higher salvage value) mitigates the demand loss driven by a high customer-accessible inventory level. Hence, with higher disposal flexibility, the firm can set higher order-up-to levels and sales prices to achieve higher profits.

5.2. Without Inventory Disposal

The model without inventory disposal applies to the cases wherein the inventory is too expensive or too inconvenient

to dispose of. For example, in the automobile industry, it is too costly to dispose of unsold cars of the last year model. In other industries, such as chemical engineering, products are often so environmentally unfriendly that they cannot be disposed of arbitrarily. The model without inventory disposal has a simpler optimal policy structure (customer-accessible-inventory-dependent order-up-to/display-up-to list-price policy) and, like the model without inventory withholding, delivers sharper insights about the impacts of inventory-dependent demand and inventory withholding policy. More specifically, we show that inventory-dependent demand motivates the firm to order less and charge a lower sales price, whereas the inventory withholding policy helps to mitigate the overage risk and increases the optimal order-up-to levels and sales prices.

As a counterpart to Theorem 4, the following theorem shows that inventory-dependent demand drives down the optimal order-up-to levels and sales prices in the model without inventory disposal:

THEOREM 6. *Consider a model without inventory disposal. For any $t = T, T-1, \ldots, 1$, assume that $r_d = r_w = 0$, and $h_w \geqslant h_a$, i.e., reallocation is cost free and it is more costly to store the inventory in the warehouse. In addition, assume that $D_t = \delta(d_t, I_t^a, \epsilon_t)$ and $\hat{D}_t = \hat{\delta}(d_t, I_t^a, \epsilon_t)$ with inventory dependent term $\gamma(I_t^a)$ and $\hat{\gamma}(I_t^a)$, respectively, where $\hat{\gamma}(I_t^a) = \gamma_0 = \lim_{x \to -\infty} \gamma(x)$ for all $I_t^a \leqslant K_a$, i.e., $\hat{D}_t$ does not depend on the customer-accessible inventory level. Further assume that the demand is of additive form (i.e., $\epsilon_t^m = 1$ with probability 1). We have*:

(a) *The firm in the system with demand $\hat{D}_t$ should not withhold any inventory.*

(b) $x_t^a(I_t^a) \leqslant \hat{x}_t^{s*}(I_t^a)$ and $d_t(I_t^a) \geqslant \hat{d}_t^{s*}(I_t^a)$ for all $I_t^a \leqslant K_a$.

An inventory withholding policy enables the firm to better control demand by intentionally making part of its inventory unavailable to its customers. Hence, an inventory withholding policy can stabilize the demand process and increase the optimal order-up-to levels and sales prices, as shown below:

THEOREM 7. *Consider a model without inventory disposal. For any $t = T, T-1, \ldots, 1$, assume that the demand is of additive form (i.e., $\epsilon_t^m = 1$ with probability 1), $r_d = r_w = 0$ (i.e., reallocation is cost free). If $I_t = I_t^a$, we have $x_t^a(I_t^a) \geqslant x_t^{s*}(I_t^a)$ for $I_t^a \leqslant \max\{I_t^a : x_t^a(I_t^a) \geqslant I_t^a\}$, and $d_t^*(I_t^a, I_t) \leqslant d_t^{s*}(I_t^a)$ for $I_t^a \leqslant K_a$.*

Note that Theorem 6 needs the assumption that inventory reallocation is cost free ($r_d = r_w = 0$); this assumption is necessary to reduce the state space dimension in its proof. We also assume $r_d = r_w = 0$ for Theorem 7, mainly for expositional convenience. The results still hold under the general condition that $r_d, r_w \geqslant 0$.

To summarize, inventory withholding and inventory disposal have similar strategic implications in addressing inventory-dependent demand. The firm uses these strategies to hedge against the overage risk caused by the scarcity effect of inventory and stimulate more potential demand.

6. Responsive Inventory Reallocation

In our previous analysis, we assume that the firm can withhold and redisplay inventory only at the beginning of the decision epoch before the demand is realized. We now relax this assumption by allowing the firm to responsively reallocate its on-hand inventory after the demand realization. Responsive inventory reallocation enables the firm to optimize its inventory policy after the demand uncertainty is realized, so that the supply and demand are better matched and the trade-off between meeting current and inducing potential demand is better balanced. Note that when responsive inventory reallocation is allowed, the firm should not reallocate its inventory before the demand is realized.

At the beginning of each period, the firm chooses its inventory replenishment/disposal quantity and the sales price. The demand is then realized, after which the firm decides the inventory reallocation quantities between the warehouse and customer-accessible storage.

To formulate the planning problem as a dynamic program, let

$V_t^r(I_t^a, I_t) =$ the maximum expected discounted profits in periods $t, t-1, \ldots, 1$, when starting period t with a customer-accessible inventory level I_t^a and a total inventory level I_t,

where the superscript "r" refers to "responsive inventory reallocation." Without loss of generality, we assume that the excess inventory in the last period (period 1) is discarded without any salvage value, i.e., $V_0^r(I_0^a, I_0) = 0$, for any (I_0^a, I_0).

We first analyze the optimal reallocation policy in period t. Assume that the order-up-to/dispose-down-to level set by the firm before the demand realization is x_t and that the realized demand is D_t. The optimal display-up-to level, $x_t^{ra*}(I_t^a, x_t, D_t)$, after inventory reallocation, is given by:

$$
\begin{aligned}
x_t^{ra*}(I_t^a, x_t, D_t) = \underset{\min\{0, I_t^a - D_t\} \leqslant x_t^a \leqslant x_t - D_t}{\operatorname{argmax}} & \left\{ -r_d(x_t^a - I_t^a + D_t)^+ \right. \\
& -r_w(x_t^a - I_t^a + D_t)^- - bx_t^{a-} - h_a x_t^{a+} \\
& \left. -h_w(x_t - x_t^a - D_t) + \alpha V_{t-1}^r(x_t^a, x_t - D_t) \right\}
\end{aligned}
$$

Hence, the optimal value functions satisfy the following recursive scheme:

$$
\begin{aligned}
& V_t^r(I_t^a, I_t) \\
& = \max_{(x_t, d_t) \in F^r(I_t^a)} \left\{ p(d_t) \mathbb{E}\{\delta(p(d_t), I_t^a, \epsilon_t)\} \right. \\
& \quad - c(x_t - I_t)^+ + s(x_t - I_t)^- \\
& \quad + \mathbb{E}_{D_t} \left\{ \max_{\min\{0, I_t^a - D_t\} \leqslant x_t^a \leqslant \min\{K_a, x_t - D_t\}} \{ -r_d(x_t^a - I_t^a + D_t)^+ \right. \\
& \quad -r_w(x_t^a - I_t^a + D_t)^- - bx_t^{a-} - h_a x_t^{a+} \\
& \quad \left. \left. -h_w(x_t - x_t^a - D_t) + \alpha V_{t-1}^r(x_t^a, x_t - D_t)\} \right\} \right\},
\end{aligned}
$$

where $F^r(I_t^a) := \{(x_t, d_t) : x_t \geqslant \min\{I_t^a, 0\}, d_t \in [\underline{d}, \bar{d}]\}$. Following the algebraic manipulation similar to that in Equation (5), we have:

$$V_t^r(I_t^a, I_t)$$

$$= r_d I_t^a + c I_t + \max_{(x_t, d_t) \in F^r(I_t^a)} \left\{ R(d_t, I_t^a) + r_d(d_t + \gamma(I_t^a)) \right.$$

$$- \theta(x_t - I_t)^- - \psi x_t$$

$$+ \mathbb{E}_{D_t} \left\{ \max_{\min\{D_t, I_t^a\} \leqslant y_t^a \leqslant \min\{x_t, K_a + D_t\}} \{-(r_d + r_w)(y_t^a - I_t^a)^- \right.$$

$$\left. \left. + \phi y_t^a + G_t^r(y_t^a - D_t, x_t - D_t)\} \right\} \right\}, \tag{12}$$

with $G_t^r(x, y) := -(h_a + b)x^+$

$$+ \alpha[V_{t-1}^r(x, y) - r_d x - c y].$$

Comparing the value functions (12) and (4), it is clear that by postponing the reallocation decision until after demand realization, the firm achieves a higher expected total profit. In the following theorem, we characterize the optimal inventory replenishment/disposal/reallocation and pricing policy in the model with responsive inventory reallocation:

Theorem 8. *The following statements hold for $t = T, T - 1, \dots, 1$:*

(a) *$V_t^r(I_t^a, I_t)$ is jointly concave and continuously differentiable in (I_t^a, I_t), whereas the normalized value function $V_t^r(I_t^a, I_t) - r_d I_t^a - c I_t$ is decreasing in I_t^a and I_t.*

(b) *For any given x_t and realized D_t, $v_t^r(y_t^a | I_t^a, x_t, D_t) := -(r_d + r_w)(y_t^a - I_t^a)^- + \phi y_t^a + G_t^r(y_t^a - D_t, x_t - D_t)$ is concave in y_t^a. Therefore, the optimal customer-accessible-inventory level is:*

$$x_t^{ra*}(I_t^a, x_t, D_t)$$

$$= \underset{\min\{D_t, I_t^a\} \leqslant y_t^a \leqslant \min\{x_t, K_a + D_t\}}{\operatorname{argmax}} \{v_t^r(y_t^a | I_t^a, x_t, D_t)\} - D_t.$$

(c) *There exist two customer-accessible inventory-level-dependent thresholds, $x_t^r(I_t^a)$ and $\tilde{x}_t^r(I_t^a)$ $(x_t^r(I_t^a) \leqslant \tilde{x}_t^r(I_t^a))$, such that it is optimal to order up to $x_t^r(I_t^a)$ if and only if $I_t < x_t^r(I_t^a)$, to dispose down to $\tilde{x}_t^r(I_t^a)$, if and only if $I_t > \tilde{x}_t^r(I_t^a)$, and to keep the total inventory level otherwise. Moreover, there exist two customer-accessible-inventory-level-dependent sales prices, $p(d_t^r(I_t^a))$ and $p(\tilde{d}_t^r(I_t^a))$, such that it is optimal to charge a sales price $p(d_t^r(I_t^a))$ if $I_t \leqslant x_t^r(I_t^a)$, and to charge a sales price $p(\tilde{d}_t^r(I_t^a))$ if $I_t \geqslant \tilde{x}_t^r(I_t^a)$.*

Theorem 8(a) proves the joint concavity and continuous differentiability of the optimal value functions. Part (b) shows that, in each period, the optimal reallocation policy is obtained by solving a one-dimensional convex optimization after the demand is realized. Consistent with Theorem 1, part (c) of Theorem 8 proves that it is optimal to

order if the total inventory level is low ($I_t < x_t^r(I_t^a)$), to dispose of inventory if it is high ($I_t > \tilde{x}_t^r(I_t^a)$), and to maintain the starting inventory level otherwise. Compared with Theorem 1, which characterizes optimal policy in the unified model, Theorem 8 demonstrates that it is possible that the firm orders and withholds some inventory under the optimal responsive inventory reallocation policy, because, in this case, the firm has the flexibility to reallocate inventory after the demand uncertainty is resolved.

As in Theorem 2, we can show that if the warehouse holding cost, h_w, is high enough, it is optimal not to hold any inventory in the warehouse; if the salvage value, s, is low enough, it is optimal not to dispose of anything; and if the reallocation fee to withhold inventory, r_w, is high enough, it is optimal not to reallocate any customer-accessible inventory to the warehouse.

7. Numerical Studies

This section reports a set of numerical studies that (a) verify the robustness of our analytical results when Assumption 3 does not hold; (b) quantify the profit loss of ignoring the scarcity effect of inventory when making the pricing and inventory decisions; and (c) quantitatively evaluate the benefit of dynamic pricing in the presence of the scarcity effect. Our numerical results demonstrate that (1) the structural results developed in our theoretical model are robust and hold for a large set of nonconcave $R(\cdot, \cdot)$ functions; (2) the impact of the scarcity effect is significant and is higher when the scarcity intensity, demand variability, and/or planning horizon length increase; and (3) the value of dynamic pricing under the scarcity effect is significant and higher under higher scarcity intensity, demand variability, and/or shorter planning horizon.

Throughout our numerical studies, we assume that the firm can neither withhold nor dispose of its on-hand inventory for two reasons: (a) to have a clear illustration of the optimal policy structure in a model where Assumption 3 does not hold; and (b) to single out and highlight the impact of the focal operational elements (i.e., the scarcity effect of inventory and the dynamic pricing strategy). We also assume that the demand in each period is of the additive form, i.e., $\epsilon_t^m = 1$ almost surely and $D_t = d_t + \gamma(I_t^a) + \epsilon_t^a$. Let $\{\epsilon_t^a\}_{t=1}^T$ follow *i.i.d.* normal distributions with mean 0 and standard deviation σ. The inverse demand function is linear with slope -1, i.e., $p(d_t) = p_0 - d_t$. We set the discount factor $\alpha = 0.95$, the unit holding cost $h = 1$, and the unit backlogging cost $b = 10$.

7.1. Optimal Policy Structure with Nonconcave $R(\cdot, \cdot)$ Functions

In this subsection, we numerically examine whether the structural results in our theoretical model are robust when Assumption 3 does not hold, i.e., $R(\cdot, \cdot)$ is not jointly concave. We have performed extensive numerical experiments

to test the robustness of our analytical results. In all our numerical experiments, although Assumption 3 is violated, the characterizations of the optimal policy by our theoretical analysis (i.e., Theorem 1, Theorem 3, and Theorem 4) continue to hold. More specifically, our numerical results verify that (a) the inventory-dependent order-up-to/list-price policy is optimal and the order-up-to level is decreasing in the starting inventory level; (b) the optimal sales price [price-induced demand] is decreasing [increasing] in the starting inventory level; and (c) compared to an inventory system without the scarcity effect, the firm with the scarcity effect sets lower order-up-to levels and lower sales prices. Therefore, the structural results of our theoretical model are robust and hold for nonconcave $R(\cdot, \cdot)$ functions in all our numerical experiments.

Note that from Lemma 2(a) if the scarcity function $\gamma(\cdot)$ contains a linear and strictly decreasing piece, $R(\cdot, \cdot)$ is not jointly concave. Hence, we report our numerical results for the case wherein

$$
\gamma(I_t^a) = \begin{cases} \gamma_0 - \exp(\eta I_t^a), & \text{for } I_t^a \leqslant 0, \\ \gamma_0 - 1 - \eta I_t^a, & \text{for } 0 < I_t^a \leqslant K_a, \end{cases} \quad \text{with } \eta > 0.
$$

It is clear that $\gamma(\cdot)$ is concavely decreasing and continuously differentiable in I_t^a for all $I_t^a \leqslant K_a$, but $R(\cdot, \cdot)$ is not jointly concave in the region $\{(d_t, I_t^a): d_t \in [\underline{d}, \bar{d}], I_t^a \in [0, K_a]\}$. We have performed extensive numerical experiments that test many combinations of different values of p_0, γ_0, c, η, σ, $\underline{d}$, $\bar{d}$, K_a, and t. In all the scenarios we examine, the predictions of the optimal policy by our theoretical analysis (i.e., Theorem 1, Theorem 3, and Theorem 4) continue to hold without Assumption 3. Figures 1–2 illustrate the optimal order-up-to level and price-induced demand with the parameter values $p_0 = 30$, $\gamma_0 = 9$, $c = 8$, $\eta = 0.5$, $\sigma = 2$, $[\underline{d}, \bar{d}] = [6, 12]$, $K_a = 18$, and $t = 20$.

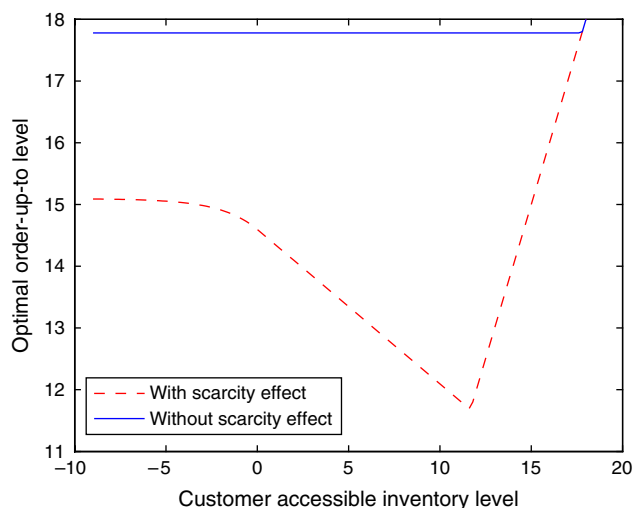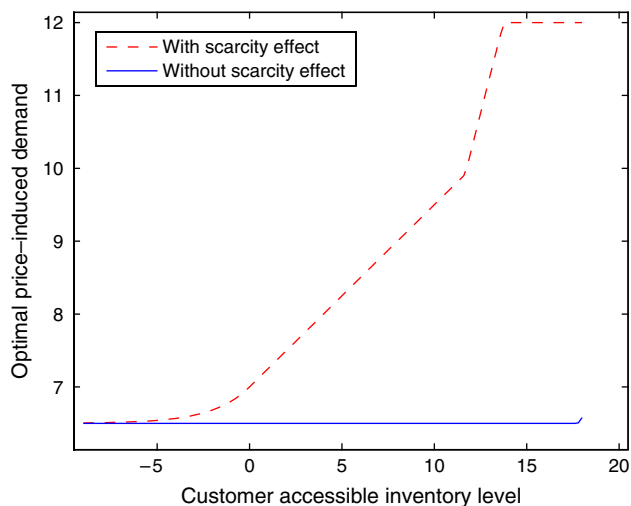**Figure 1.** (Color online) Optimal ordering-up-to level.

Figure 2. (Color online) Optimal price-induced demand.

7.2. Impact of Scarcity Effect

In this subsection we numerically study the impact of the scarcity effect of inventory on the firm's profitability by quantifying the profit loss of ignoring this effect under different levels of scarcity effect intensity, demand variability, and planning horizon length. As in §7.1, we assume that

$$
\gamma(I_t^a) = \begin{cases} \gamma_0 - \exp(\eta I_t^a), & \text{for } I_t^a \leqslant 0, \\ \gamma_0 - 1 - \eta I_t^a, & \text{for } 0 < I_t^a \leqslant K_a, \end{cases} \quad \text{where } \eta > 0.
$$

Note that η represents the scarcity effect intensity of the inventory system: The larger the η, the more intense the scarcity effect. We need to evaluate the profit of a firm that ignores the scarcity effect, $\tilde{V}$. To compute $\tilde{V}$, we first numerically obtain the optimal policy in an inventory system without the scarcity effect and then evaluate the total profits of this policy in an inventory system with the scarcity effect. We also evaluate the optimal profit of a firm under the scarcity effect, V^*. In the evaluation of V^* and $\tilde{V}$, we take $I_t^a = 0$ as the reference customer-accessible inventory level. The metric of interest is

$$
\lambda_{\text{scarcity}} := \frac{V^* - \tilde{V}}{V^*}, \quad \text{under different values of } \eta, \sigma \text{ and } t.
$$

Our numerical experiments are conducted under the following values of parameters: $p_0 = 21$, $\gamma_0 = 4$, $c = 4$, $\eta = 0.35, 0.4, 0.45, 0.5, 0.55$, $\sigma = 1, 2, 3$, $[\underline{d}, \bar{d}] = [6, 12]$, $K_a = 18$, and $t = 5, 10$.
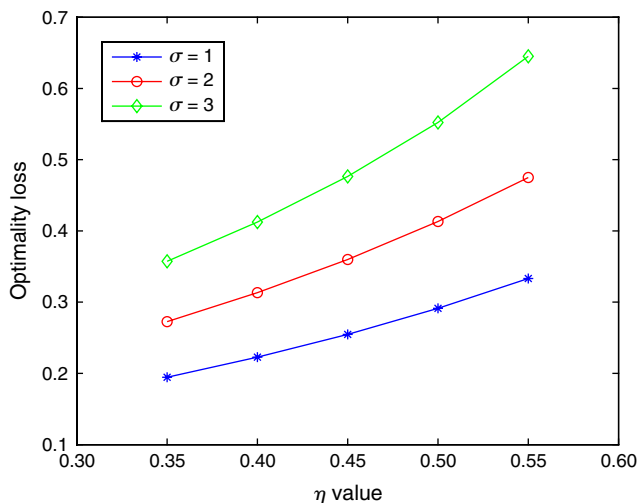
Figures 3–4 summarize the results of our numerical study on the impact of the scarcity effect on the firm's profitability. Our results show that, when the scarcity effect is ignored, all numerical experiments exhibit a significant profit loss, which is at least 16.41% and can be as high as 64.52%. Moreover, the impact of the scarcity effect is increasing in the scarcity intensity, demand variability, and

Figure 3. (Color online) Value of $\lambda_{\text{scarcity}}$: $t = 5$.
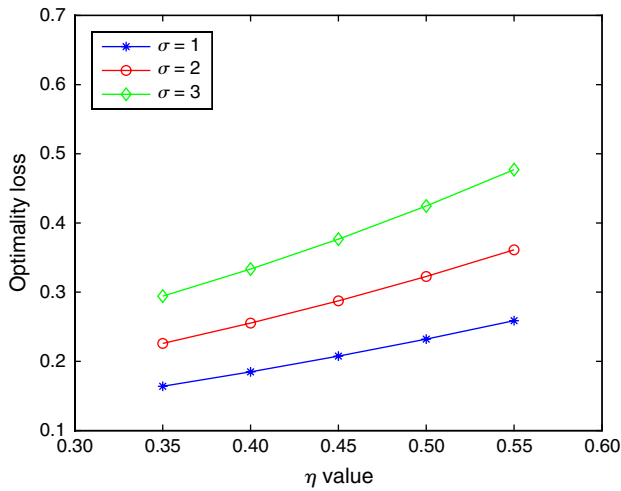
planning horizon length. The *scarcity effect* has two effects on the firm's profitability: (a) it decreases future demand, and (b) it increases demand variability because the variability of potential demand is intensified by that of the past demand via the *scarcity effect*. Hence, the first [second] effect lowers firm profits with higher scarcity intensity [demand variability]. The comparison between Figures 3–4 implies that the impact of the scarcity effect accumulates over time, so that the profit loss of ignoring the scarcity effect is higher under a longer planning horizon. In short, the scarcity effect of inventory matters significantly to the firm's profitability when the scarcity effect intensity and demand variability are high, and the planning horizon is long. Our numerical finding confirms the result in Sapra et al. (2010) that the profit loss is increasing in the scarcity effect intensity. On the other hand, our numerical finding on the impact of demand variability contrasts that in Sapra

et al. (2010), which shows that the profit loss of ignoring the scarcity effect is decreasing in demand variability. In their experiments, the potential demand is convexly decreasing in the leftover inventory level, so higher demand variability increases the expected potential demand and, thus, the firm's profitability under the scarcity effect.

7.3. Value of Dynamic Pricing

In this subsection, we numerically explore the value of dynamic pricing under the scarcity effect of inventory with different levels of scarcity effect intensity, demand variability, and planning horizon length. As in §§7.1–7.2, we assume that

$$\gamma(I_t^a) = \begin{cases} \gamma_0 - \exp(\eta I_t^a), & \text{for } I_t^a \leqslant 0, \\ \gamma_0 - 1 - \eta I_t^a, & \text{for } 0 < I_t^a \leqslant K_a, \end{cases} \quad \text{where } \eta > 0.$$

We evaluate the profit of a firm that adopts the optimal static pricing strategy, $\hat{V}$. To compute $\hat{V}$, we first evaluate the total profit of an inventory system for any fixed price p_t in each t, and then maximize over p_t to select the optimal static price. Consistent with V^*, $\hat{V}$ is evaluated at the reference customer-accessible inventory level $I_t^a = 0$. The metric of interest is

$$\lambda_{\text{pricing}} := \frac{V^* - \hat{V}}{\hat{V}}, \quad \text{under different values of } \eta, \sigma \text{ and } t.$$

Our numerical experiments are conducted under the following values of parameters: $p_0 = 21$, $\gamma_0 = 4$, $c = 4$, $\eta = 0.35$, $0.4, 0.45, 0.5, 0.55$, $\sigma = 1, 2, 3$, $[\underline{d}, \bar{d}] = [6, 12]$, $K_a = 18$, and $t = 5, 10$.

Figures 5–6 summarize the results of our numerical study on the value of dynamic pricing. The results show that the value of dynamic pricing is significant in the presence of the scarcity effect. Federgruen and Heching (1999)
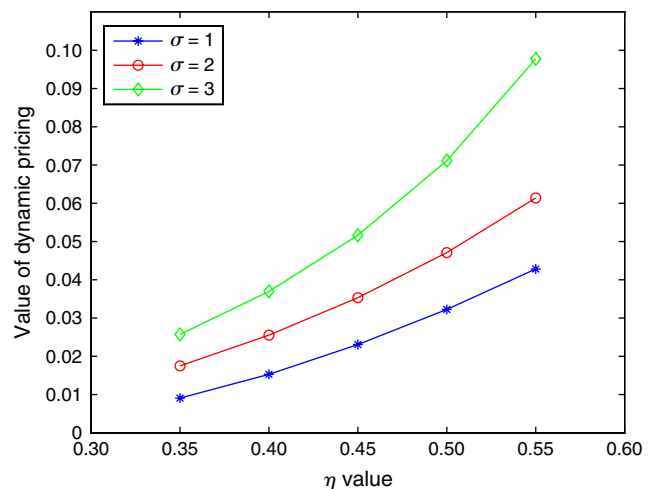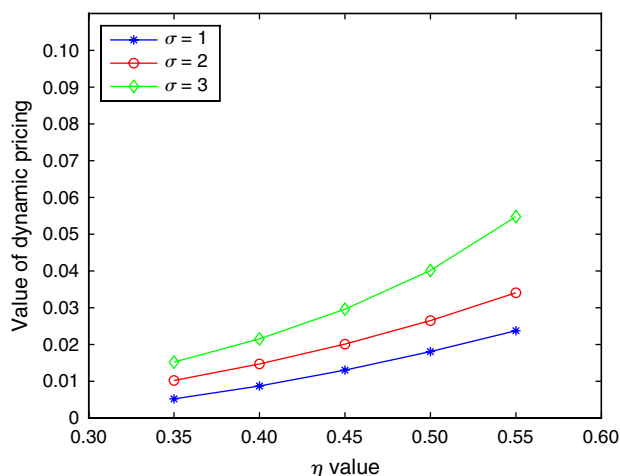
Figure 4. (Color online) Value of $\lambda_{\text{scarcity}}$: $t = 10$.

Figure 5. (Color online) Value of λ_{pricing}: $t = 5$.

Figure 6. (Color online) Value of λ_{pricing}: $t = 10$.

document that the profit improvement of dynamic pricing in a 5-period model is between 0.46%–2.24%, when the coefficient of variation for demand varies between 0.7 and 1.4. The numerical experiments of Figure 5 report a much higher profit improvement (between 0.91%–9.78%) of dynamic pricing in a 5-period model with the coefficient of variation of demand between 0.11 and 0.33. Thus, the scarcity effect of inventory gives rise to significantly higher value of dynamic pricing. Dynamic pricing value is driven by the following three effects: (a) it achieves a better match between supply and demand; (b) it helps induce higher future demand; and (c) it dampens future demand variability. While effect (a) also improves the performance of an inventory system without the scarcity effect, effects (b) and (c) have their impact only on a firm with the scarcity effect. Therefore, the value of dynamic pricing is significantly increased by the scarcity effect. Moreover, with higher scarcity effect intensity [demand variability], effects (b) and (c) enhance the firm's profitability more significantly. The comparison between Figures 5 and 6 implies that the value of dynamic pricing decreases over time. This is consistent with the findings in Federgruen and Heching (1999) that the optimal dynamic pricing policy converges to the optimal static pricing policy, as the planning horizon length goes to infinity. In short, the value of dynamic pricing under the scarcity effect of inventory is most significant when the intensity of scarcity effect and demand variability is high, and the planning horizon length is moderate.

To summarize, we note that all the numerical results and insights in this section are robust and hold for (a) the general demand form, Equation (1), and (b) a large variety of different inverse demand functions (i.e., $p(\cdot)$) and scarcity functions (i.e., $\gamma(\cdot)$) that give rise to concave or nonconcave $R(\cdot, \cdot)$ functions.

8. Concluding Remarks

We conclude this paper with a summary of the main results and managerial insights derived from our model and some thoughts on a possible direction for future research.

8.1. Summary

To our knowledge, this paper is the first in the literature to study the joint pricing and inventory management model under the scarcity effect of inventory. Demand is modeled as a decreasing stochastic function of price and customer-accessible inventory level. We propose a unified model in which the firm has several operational options to hedge against the risk of the stochastic inventory-dependent demand: (a) dynamic pricing, through which the firm can dynamically adjust its sales price; (b) inventory withholding, through which the firm can withhold part of its inventory from customers; and (c) inventory disposal, through which the firm can dispose of part of its surplus inventory. We show that a customer-accessible inventory-dependent order-up-to/dispose-down-to/display-up-to list-price policy is optimal. The order-up-to/display-up-to and list price levels are decreasing in the customer-accessible inventory level because of the negative dependence of demand on inventory. When the scarcity effect of inventory is sufficiently strong, the firm can strategically benefit from the scarcity effect by displaying no positive inventory and making every customer wait; the revenue generated by the strong scarcity effect dominates the backlogging cost of the wait list.

When the warehouse holding cost [salvage value] is sufficiently high [low], it is too costly to withhold [dispose of] inventory, and the unified model is reduced to the model without inventory withholding [disposal]. The model without inventory withholding [disposal] generates additional results and sharper insights. In the model without inventory withholding/disposal, we show that optimal sales prices and order-up-to levels are lower under the scarcity effect of inventory than those under inventory-independent demand. Higher operational flexibility (a higher salvage value or the inventory withholding opportunity), however, helps the firm hedge against the overstocking risk and, hence, drives the firm to set higher order-up-to/display-up-to levels and sales prices.

In addition, responsive inventory reallocation is another effective way to address the scarcity effect of inventory. Reallocation flexibility after demand realization enables the firm to better hedge against the demand uncertainty and balance the trade-off between meeting current demand and inducing potential demand. In this case, because the firm can reallocate its on-hand inventory after demand is realized, it may be optimal to order and withhold when the realized demand is small.

We perform extensive numerical studies to demonstrate (a) the robustness of our analytical results, (b) the impact of the scarcity effect on the profit of the firm, and (c) the value of dynamic pricing under the scarcity effect of inventory. Our numerical results show that the analytical characterizations of the optimal policies in our model are robust and hold for nonconcave $R(\cdot, \cdot)$ functions in all our experiments. The impact of the scarcity effect on the firm's profit is two-fold: (a) it decreases future demand; and

(b) it increases demand variability. Hence, the profit loss of ignoring the scarcity effect is higher under higher scarcity intensity (via effect (a)), higher demand variability (via effect (b)), and longer planning horizon (via both effects). The value of dynamic pricing under the scarcity effect is three-fold: (a) it better matches supply and demand; (b) it helps induce higher future demand; and (c) it dampens future demand variability. Effects (b) and (c) lead to higher value of dynamic pricing under higher scarcity intensity [demand variability]. Moreover, the optimal dynamic pricing policy converges to the optimal static pricing policy as the planning horizon length goes to infinity, so that the value of dynamic pricing decreases over time.

Finally, we note that all the analytical results in this paper can be easily extended to the infinite horizon discounted model with the standard argument that demonstrates the preservation of the structural properties as the planning horizon length goes to infinity.

8.2. Extension

In this subsection, we propose a possible extension of our work: analysis of the model that encompasses both the scarcity effect and the *promotional effect* of inventory.

As discussed in §2, the displayed inventory has the service and the promotional effects (see, e.g., Balakrishnan et al. 2004 and 2008) because a higher customer-accessible inventory level creates a stronger visual impact and customers infer a greater chance to obtain the product. In the literature, this phenomenon is also called the billboard effect and the *shelf-space effect* (e.g., Cachon et al. 2013, Baron et al. 2011 and Chen et al. 2012).

It is interesting to analyze the model that incorporates both the scarcity effect of prereplenishment inventory and the promotional effect of post-replenishment inventory. More specifically, we assume that the demand in period t, $D_t = \delta(p_t, I_t^a, x_t^a, \epsilon_t) = (d(p_t) + \gamma_1(I_t^a) + \gamma_2(x_t^a))\epsilon_t^m + \epsilon_t^a$, where $\gamma_1(\cdot)$ is a decreasing function of prereplenishment customer-accessible inventory level I_t^a, and $\gamma_2(\cdot)$ is an increasing function of post-replenishment customer-accessible inventory level x_t^a. As before, assume that $d(\cdot)$ is a strictly decreasing function of sales price p_t, $\mathbb{E}\{\epsilon_t^m\} = 1$ and $\mathbb{E}\{\epsilon_t^a\} = 0$.

It is challenging to characterize the optimal joint pricing and inventory management policy under this generalized inventory-dependent demand. In particular, the effect of inventory on the firm's profitability is more involved and it is unclear how to strike a balance between the overage and underage risks in this model. We will explore this problem in our future research.

Supplemental Material

Supplemental material to this paper is available at http://dx.doi.org/10.1287/opre.2014.1306.

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